From the force diagram the reactions at P, Q are equal, hence either the major axis is vertical or the minor axis is vertical, giving the other two positions of equilibrium.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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156. Proposed by A. H. HOLMES, Brunswick, Maine.

Find integral values for a, b, c, d, and e in the equation, $a^2+b^2+c^2+d^2=e^2$.

I. Solution by the PROPOSER.

Take $a=32(40^2-9^2)$; $b=64(40\times9)$; $c=24(40^2+9^2)$; $d=9(40^2+9^2)$. Then $a^2=32^2\times1519^2$; $b^2=32^2\times720^2$; $c^2=24^2\times1681^2$; $d^2=81\times1681^2$. And $e^3=32^2[1519^2+720^2+1681^2(24^2+81)]$.

$\therefore a=48608$; $b=23040$; $c=40344$; $d=15129$; $e=1681$.

In the above problem it was intended by the author that e^2 should be e^3. ED. F.

II. Solution by ARTEMAS MARTIN, LL. D., Coast Survey Office, Washington, D. C.

In the well-known identity

$$(x-y)^2+4xy=(x+y)^2, \tag{1}$$

x and y may have any values whatever, and we can, therefore, assume $x=u+v+w$; then (1) becomes

$$(u+v+w-y)^2+4y(u+v+w)=(u+v+w+y)^2. \tag{2}$$

Now take $u=p^2$, $v=q^2$, $w=r^2$, $y=s^2$; then (2) becomes

$$(p^2+q^2+r^2-s^2)^2+(2ps)^2+(2qs)^2+(2rs)^2=(p^2+q^2+r^2+s^2)^2, \quad (3);$$

and p, q, r, s may be any values chosen at pleasure.

Therefore we may take $a=p^2+q^2+r^2+s^2$, $b=2ps$, $c=2qs$, $d=2rs$, $e=p^2+q^2+r^2+s^2$, or in any other order we please.

Examples. 1. Take $p=1$, $q=2$, $r=3$, $s=1$; then $2^2+4^2+6^2+13^2=15^2$.

2. Take $p=1$, $q=2$, $r=3$, $s=4$; then, after dividing through by 2^2 and discarding the negative sign, $1^2+4^2+8^2+12^2=15^2$.

3. Take $p=2$, $q=3$, $r=4$, $s=5$; then, after dividing through by 2^2, $2^2+10^2+15^2+20^2=27^2$.

See *Mathematical Magazine*, Vol. II, No. 5, pp. 69-76; No. 6, pp. 89-96; No. 8, pp. 137-140; and No. 11, pp. 209-220, for various methods of finding many sets of square numbers whose sum is a square.

III. **Solution by ARTEMAS MARTIN, LL. D., Coast Survey Office, Washington, D. C.**

Let $a=x+p$, $b=x-p$, $c=x+q$, $d=x-q$, $e=2x+n$; then by substitution and reduction, $4x^2+2p^2+2q^2$, from which

$$x=\frac{2p^2+2q^2-n^2}{4n}.$$

Substituting the value of x and rejecting the common denominator $4n$, we may take

$$a=2p^2+2q^2-n^2+4np,$$
$$b=2p^2+2q^2-n^2-4np,$$
$$c=2p^2+2q^2-n^2+4nq,$$
$$d=2p^2+2q^2-n^2-4nq,$$
$$e=4p^2+4q^2+2n^2.$$

Examples. 1. Take $p=3$, $q=2$, $n=1$; then we have $13^2+17^2+33^2+37^2=54^2$.

2. Take $p=1$, $q=2$, $n=3$, and we have $11^2+13^2+23^2+25^2=38^2$.

3. Take $p=3$, $q=4$, $n=2$, and we get $7^2+11^2+35^2+39^2=54^2$.

Also solved by V. M. Spunar, S. E. Corey, G. B. M. Zerr, and J. Scheffer.

NOTES AND NEWS.

This first number of the sixteenth volume of the MONTHLY has been delayed for various reasons, including the culmination of plans whereby its future publication is to be under the joint auspices of The University of Chicago and the University of Illinois. In accordance with this plan, Professor G. A. Miller, of the University of Illinois, and Professor H. E. Slaught, of the University of Chicago, will share jointly the editorial responsibility of articles. Professor B. F. Finkel, of Drury College, will continue in charge of the problem department and have general management of the MONTHLY, as heretofore, and Professor L. E. Dickson, of The University of Chicago, whose editorial cooperation has been of the highest value, will retire from active service. It is believed that the union of these two universities in the interests of the MONTHLY will result in the further development of its usefulness and in the extension of its influence in its particular field, namely, the realm of college mathematics as distinguished, on the one hand, from that of the secondary schools, and on the other hand, from that of the graduate schools.